Multiplicative Operations of Vectors and Scalars

Consider a scalar quantity a and a vector quantity **B**. We express the multiplication of these two values as:

$$a\mathbf{B} = \mathbf{C}$$

In other words, the product of a scalar and a vector—is a **vector**!

Q: *OK, but what **is** vector **C** ? What is the **meaning** of a **B** ?*

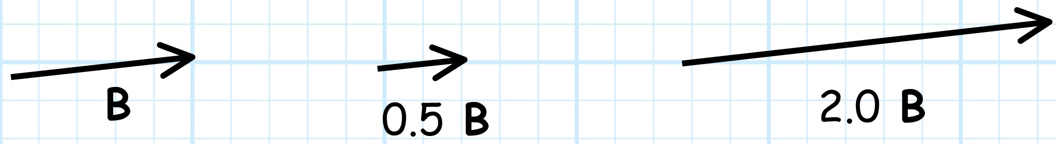
A: The resulting vector **C** has a **magnitude** that is equal to a times the **magnitude** of **B**. In other words:

$$|\mathbf{C}| = a\,|\mathbf{B}|$$

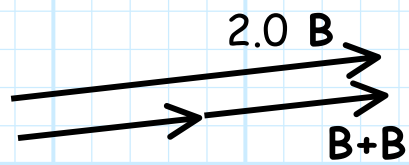
However, the **direction** of vector **C** is **exactly** that of **B**.

Therefore multiplying a vector by a scalar changes the **magnitude** of the vector, but **not** its direction.

For example:



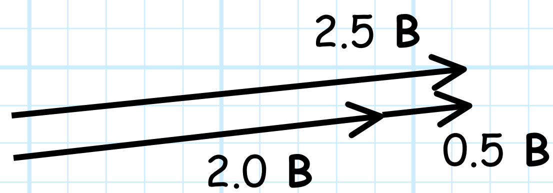
Note that $\mathbf{B} + \mathbf{B} = 2.0\,\mathbf{B}$!



1. More generally, we find that scalar-vector multiplication is **distributive** as:

$$a\,\mathbf{B} + b\,\mathbf{B} = (a+b)\,\mathbf{B}$$

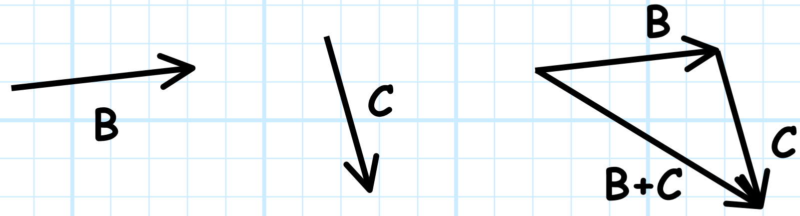
E.G.,

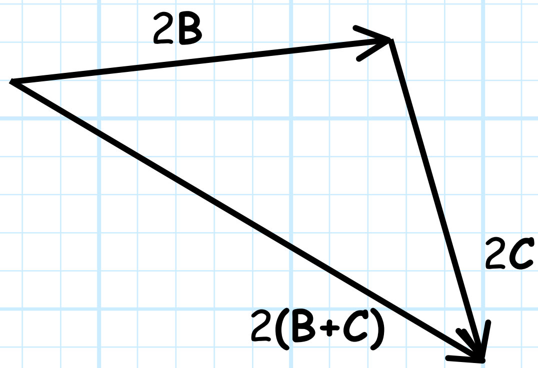


2. And **also distributive** as:

$$a\,\mathbf{B} + a\,\mathbf{C} = a\,(\mathbf{B} + \mathbf{C})$$

E.G.,





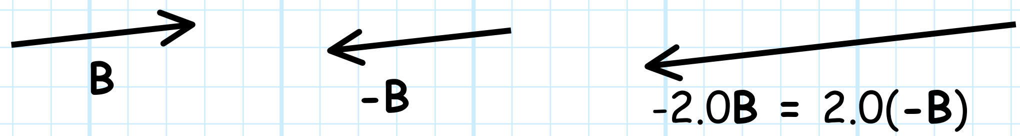
3. Scalar-Vector multiplication is also **commutative**:

$$a\,\mathbf{B} = \mathbf{B}\,a$$

4. Multiplication of a vector by a **negative** scalar is interpreted as:

$$-a\,\mathbf{B} = a\,(-\mathbf{B})$$

E.G.,



5. **Division** of a vector by a scalar is the same as multiplying the vector by the **inverse** of the scalar:

$$\frac{\mathbf{B}}{a} = \left(\frac{1}{a}\right)\mathbf{B}$$

Scalar-Vector multiplication is likewise used in many **physical** applications. For example, say you start in Lawrence and head **west** at **70 mph** for exactly **3.3 hours**.

Note your velocity has both **direction** (west) and **magnitude** (70 mph) – it's a **vector**! Lets denote it as **V** = 70 mph west.

Likewise, your travel time is a **scalar**; lets denote it as *t* = 3.3 h.

Now, lets **multiply** the two together (i.e., *t* **V**). The **magnitude** of the resulting vector is 70(3.3) = **231 miles**. The **direction** of the resulting vector is of course **unchanged**: west.

A vector describing a distance and a direction—a **directed distance**! We find that $t\,\mathbf{V} = \overline{\mathbf{R}}$, where $\overline{\mathbf{R}}$ identifies your **location** after 3.3 hours!